

Structure of Atom

Particle Nature of Electromagnetic Radiation: Planck's Quantum Theory

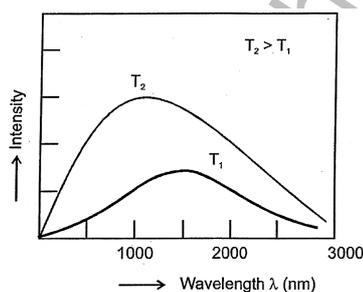
Some of the experimental phenomenon such as diffraction and interference can be explained by the wave nature of the electromagnetic radiation. However, following are some of the observations which could not be explained

- the nature of emission of radiation from hot bodies (black - body radiation)
- ejection of electrons from metal surface when radiation strikes it (photoelectric effect)

Black Body Radiation:

When solids are heated they emit radiation over a wide range of wavelengths.

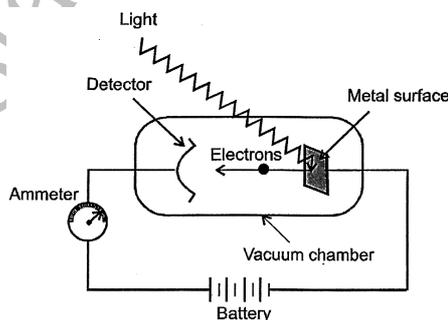
The ideal body, which emits and absorbs all frequencies, is called a **black body** and the radiation emitted by such a body is called black body radiation. The exact frequency distribution of the emitted radiation (i.e., intensity versus frequency curve of the radiation) from a black body depends only on its temperature.



The above experimental results cannot be explained satisfactorily on the basis of the wave theory of light. Planck suggested that atoms and molecules could emit (or absorb) energy only in discrete quantities and not a continuous manner.

Photoelectric Effect:

When certain metals (for example Potassium, Rubidium, Caesium etc.) were exposed to a beam of light electrons were ejected as shown in Figure.



- The electrons are ejected from the metal surface as soon as the beam of light strikes the surface, i.e., there is no time lag between the striking of light beam and the ejection of electrons from the metal surface.
- The number of electrons ejected is proportional to the intensity of brightness of light.
- For each metal, there is a characteristic minimum frequency ν_0 (also known as **threshold frequency**) below which photoelectric effect is not observed. At a frequency $\nu > \nu_0$, the ejected electrons come out with certain kinetic energy. The

kinetic energies of these electrons increases with the increase of frequency of the light used.

When a photon of sufficient energy strikes an electron in the atom of the metal, it transfers its energy instantaneously to the electron during the collision and the electron is ejected without any time lag or delay. Greater the energy possessed by the photon, greater will be transfer of energy to the electron and greater the kinetic energy of the ejected electron. In other words, kinetic energy of the ejected electron is proportional to the frequency of the electromagnetic radiation. Since the striking photon has energy equal to $h\nu$ and the minimum energy required to eject the electron is $h\nu_0$ (is also called work function, W_0) then the difference in energy ($h\nu - h\nu_0$) is transferred as the kinetic energy of the photoelectron. Following the conservation of energy principle, the kinetic energy of the ejected electron is given by the equation

$$h\nu = h\nu_0 + \frac{1}{2} m_e v^2$$

where m_e is the mass of the electron and v is the velocity associated with the ejected electron.

Example 1.

The threshold frequency ν_0 for a metal is $6 \times 10^{14} \text{ s}^{-1}$. Calculate the kinetic energy of an electron emitted when radiation of frequency $\nu = 1.1 \times 10^{15} \text{ s}^{-1}$ hits the metal.

Sol.
$$\text{K.E.} = \frac{1}{2} m_e v^2 = h(\nu - \nu_0)$$

$$\therefore \text{K.E.} = (6.626 \times 10^{-34})(1.1 \times 10^{15} - 6 \times 10^{14})$$

$$\therefore \text{K.E.} = (6.626 \times 10^{-34})(5 \times 10^{14})$$

$$= 3.313 \times 10^{-19} \text{ J}$$

Example 2.

When electromagnetic radiation of wavelength 310 nm fall on the surface of sodium, electrons are emitted with K.E. = 1.5 eV. Determine the work function (W_0) of Sodium.

Sol.
$$h\nu = \frac{12400}{3100} = 4\text{eV}, \quad \frac{1}{2} m_e v^2 = 1.5 \text{ eV}$$

$$\therefore h\nu_0 = W_0 = h\nu - \frac{1}{2} m_e v^2 = 4 - 1.5 = 2.5 \text{ eV}$$

Example 3.

When electromagnetic radiation of wavelength 300 nm falls on the surface of sodium, electrons are emitted with a kinetic energy of $1.68 \times 10^5 \text{ J mol}^{-1}$. What is the minimum energy needed to remove an electron from sodium? What is the maximum wavelength that will cause a photoelectron to be emitted?

Sol. The energy (E) of a 300 nm photon is given by $h\nu = hc/\lambda = \frac{6.626 \times 10^{-34} \text{ J s} \times 3.0 \times 10^8 \text{ ms}^{-1}}{300 \times 10^{-9} \text{ m}} = 6.626 \times 10^{-19} \text{ J}$

The energy of one mole of photons

$$= 6.626 \times 10^{-19} \times 6.022 \times 10^{23}$$

$$= 3.99 \times 10^5 \text{ J mol}^{-1}$$

The minimum energy needed to remove a mole of electron from sodium = $(3.99 - 1.68) \times 10^5 \text{ J mol}^{-1}$

$$= 2.31 \times 10^5 \text{ J mol}^{-1}$$
 The minimum energy for one electron =
$$\frac{2.31 \times 10^5 \text{ J mol}^{-1}}{6.022 \times 10^{23} \text{ electrons mol}^{-1}}$$

$$= 3.84 \times 10^{-19} \text{ J}$$

This corresponds to the wavelength $\therefore \lambda = \frac{hc}{E} = \frac{6.626 \times 10^{-34} \text{ J s} \times 3.0 \times 10^8 \text{ ms}^{-1}}{3.84 \times 10^{-19} \text{ J}} = 517 \text{ nm}$

(This corresponds to green light)

Example 4.

The threshold frequency ν_0 for a metal is $7.0 \times 10^{14} \text{ s}^{-1}$. Calculate the kinetic energy of an electron emitted when radiation of frequency $\nu = 1.0 \times 10^{15} \text{ s}^{-1}$ hits the metal.

Sol. According to Einstein's equation Kinetic energy - $\frac{1}{2} m_e v^2 = h(\nu - \nu_0) = (6.626 \times 10^{-34} \text{ Js}) (1.0 \times 10^{15} \text{ s}^{-1} - 7.0 \times 10^{14} \text{ s}^{-1})$
 $= (6.626 \times 10^{-34} \text{ Js}) \times (3.0 \times 10^{14} \text{ s}^{-1}) = 1.988 \times 10^{-19} \text{ J}$

Quantum Theory of Light:

The smallest quantity of energy that can be emitted or absorbed in the form of electromagnetic radiation is called as quantum of light.

According to Planck, the light energy coming out from any source is always an integral multiple of a smallest energy value called quantum of light.

Let quantum of light be $= E_0$ (J), then total energy coming out is $= nE_0$ ($n = \text{Integer}$)

Quantum of light = Photon (Packet or bundle of energy)

Energy of one photon is given by

$$E_0 = h\nu \quad (\nu - \text{Frequency of light})$$

$$h = 6.625 \times 10^{-34} \text{ J-Sec} \quad (h - \text{Planck const.})$$

$$E_0 = \frac{hc}{\lambda} \quad (c - \text{speed of light}), (\lambda - \text{wavelength})$$

One electron volt (e.v.) : Energy gained by an electron when it is accelerated from rest through a potential difference of 1 volt.

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ C} \times 1 \text{ volt}$$

$$\therefore 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

Example 1.

Calculate energy of one mole of photons of radiation whose frequency is $5 \times 10^{14} \text{ Hz}$.

Sol. Energy (E) of one photon is given by the expression $E = h\nu$

$$h = 6.626 \times 10^{-34} \text{ Js}$$

$$\nu = 5 \times 10^{14} \text{ s}^{-1} \text{ (given)}$$

$$E = (6.626 \times 10^{-34} \text{ Js}) \times (5 \times 10^{14} \text{ s}^{-1}) = 3.313 \times 10^{-19} \text{ J}$$

$$\text{energy of one mole of photons} = (3.313 \times 10^{-19} \text{ J}) \times (6.022 \times 10^{23} \text{ mol}^{-1}) = 199.51 \text{ KJ mol}^{-1}$$

Example 2.

A 100 watt bulb emits monochromatic light of wavelength 400 nm. Calculate the number of photons emitted per second by the bulb

Sol. Power of the bulb = 100 watt = 100 J s^{-1}

$$\text{Energy of one photon } E = h\nu = hc/\lambda = \frac{6.626 \times 10^{-34} \text{ Js} \times 3 \times 10^8 \text{ ms}^{-1}}{400 \times 10^{-9} \text{ m}} = 4.969 \times 10^{-19} \text{ J}$$

$$\text{Number of photons emitted} = \frac{100 \text{ J s}^{-1}}{4.969 \times 10^{-19} \text{ J}} = 2.012 \times 10^{20} \text{ s}^{-1}$$

Example 3.

If a charged particle having a charge of $2e$ on being accelerated by 1 volt, its K.E. will be increased by?

Sol. K.E. = $(2e) \cdot (1V)$

$$= 2 \times 1 \text{ eV}$$

$$= 2 \times 1.6 \times 10^{-19} \text{ J}$$

$$= 3.2 \times 10^{-19} \text{ J}$$

Example 4.

Number of photons emitted by a bulb of 40 watt in 1 minute with 50% efficiency will be approximately ($\lambda = 6200 \text{ \AA}$, $hc = 12400 \text{ eV \AA}$)

- (a) 7.5×10^{20} (b) 3.75×10^{20} (c) 3.75×10^{19} (d) 3.75×10^{21}

Sol. $\Delta E = \frac{12400}{6200} = 2 \text{ eV.}$

$$2 \times 1.6 \times 10^{-19} \times n = 40 \times 60 \times 0.5$$

$$n = 3.75 \times 10^{21}$$

Example 5.

Bond energy of Br_2 is 194 kJ mole^{-1} . The minimum wave number of photons required to break this bond is ($h = 6.62 \times 10^{-34} \text{ Js}$, $c = 3 \times 10^8 \text{ m/s}$)

- (a) $1.458 \times 10^{23} \text{ m}^{-1}$ (b) $1.620 \times 10^6 \text{ m}^{-1}$ (c) $4.86 \times 10^{14} \text{ m}^{-1}$ (d) $1.45 \times 10^7 \text{ m}^{-1}$

Sol. $\frac{hc}{\lambda} \times N_A = 194 \times 10^3$

$$\bar{\nu} = \frac{1}{\lambda} = \frac{194 \times 10^3}{6.63 \times 10^{-34} \times 10^8 \times 6 \times 10^{23}}$$

$$\cong 1.62 \times 10^6 \text{ m}^{-1}$$