MOTION IN A PLANE
(Kinematics in two dimensions)

VECTOR
(Addition & Subtraction)

A physical quantity which has both magnitude (numerical value) and direction; and whose addition is commutative, is called a vector.

Ex. displacement ($\vec{s}$), velocity ($\vec{v}$), acceleration ($\vec{a}$), linear momentum ($\vec{p}$), force ($\vec{f}$), electric field intensity ($\vec{E}$).

The magnitude of a vector $\vec{a}$ is $|\vec{a}|$ or a

Triangle law of addition :-

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

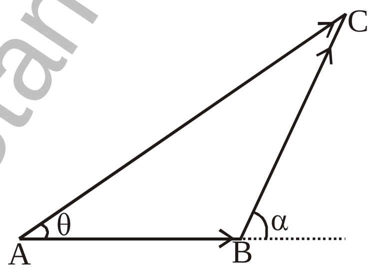
$\overrightarrow{AC}$ is resultant or vector sum of vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

If $|\overrightarrow{AB}| = P$, $|\overrightarrow{BC}| = Q$ and α is angle between $\overrightarrow{AB}$ and $\overrightarrow{BC}$,

then $\left|\overrightarrow{AC}\right| = R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$

and inclination θ of $\vec{R}$ with $\vec{P}$ is given by

$$\tan\theta = \frac{Q\sin\alpha}{P + Q\cos\alpha}$$



Special :-

1. If $\vec{P}$ & $\vec{Q}$ are in same direction ; $\alpha = 0^o$
   then $R = P + Q$ is maximum resultant
2. If $\vec{P}$ & $\vec{Q}$ are in opposite direction ; $\alpha = 180^o$
   then $R = P \sim Q$ is minimum resultant.

3. If $\vec{P}$ & $\vec{Q}$ are orthogonal (at right angles or perpendicular to each other ; $\alpha = 90^o$

then $R = \sqrt{P^2 + Q^2}$ and $\tan\theta = \frac{Q}{P}$

4. If Q = P then $R = 2P\cos\left(\frac{\alpha}{2}\right)$ and $\theta = \frac{\alpha}{2}$

the resultant $\vec{R}$ bisests the angle between then

Subtraction of vectors :-

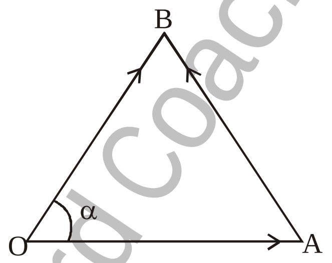
If α is angle between $\vec{P}$ and $\vec{Q}$ then angle between $\vec{P}$ and $-\vec{Q}$ is $180^o - \alpha$

If $\vec{S} = \vec{P} - \vec{Q} = \vec{P} + (-\vec{Q})$ then

$S = \sqrt{P^2 + Q^2 - 2PQ\cos\alpha}$

If Q = P then $S = 2P\sin\left(\frac{\alpha}{2}\right)$

Rotation of a vector :- If a vector of constant magnitude r turns through angle α as shown $\overrightarrow{OA} = \vec{r}_1$, $\overrightarrow{OB} = \vec{r}_2$

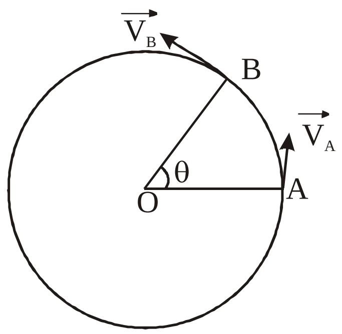


then $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \vec{r}_2 - \vec{r}_1$

$\left|\overrightarrow{AB}\right| = 2r\sin\left(\frac{\alpha}{2}\right)$

If a particle moving at constant speed v on a circle moves from position A to B as shown the change in its velocity

$\Delta\vec{V} = \overrightarrow{V_B} - \overrightarrow{V_A}$ has magnitude $\left|\Delta\vec{V}\right| = 2V\sin\left(\frac{\theta}{2}\right)$



RELATIVE MOTION

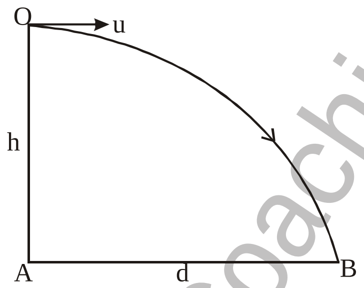
If A and B are two moving particles then velocity of A as obseved by B is called velocity of A relative to B and is given by

$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$$
$$= \vec{V}_A + \left(-\vec{V}_B\right)$$

$\vec{V}_{AB}$ is obtained by compounding (adding vectorially) the actual velocity of A with equal and opposite velocity of B.

PROJECTILES

I. A particle thrown horizontally with speed u from a point O at height h above ground moving under gravity hits the ground at B after time T.



$h = \frac{1}{2}gT^2 \qquad \therefore \quad T = \sqrt{\frac{2h}{g}}$

horizontal displacement

$d = AB = u.T$

After time t from instant of projection before reaching the ground, its velocity

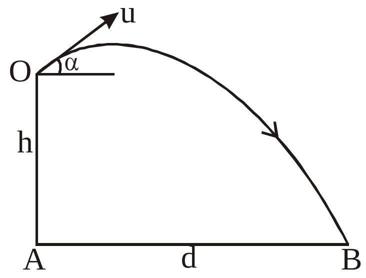
$V = \sqrt{(u)^2 + (gt)^2}$ and direction of motion is inclined at an angle θ with horizontal where $\tan\theta = \frac{gt}{u}$

horizontal displacement $x = ut$

vertical displacement $y = \frac{1}{2}gt^2$

$\therefore$ magnitude of displacement $s = \sqrt{x^2 + y^2}$

II. If a particle is thrown from point O at height h above ground with speed u making angle α above horizontal. It hits the ground at B following parabolic path after time T where

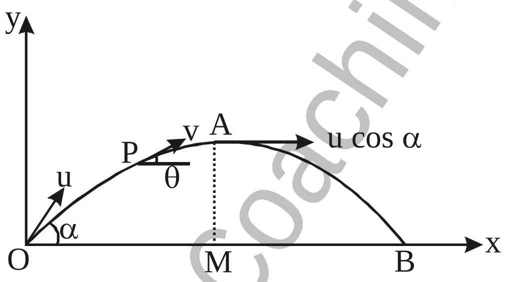


$$-h = uT\sin\alpha - \frac{1}{2}gT^2$$

the positive value of T has to be considered.

horizontal displacement $d = AB = uT\cos\alpha$

III. A particle is thrown from point O on ground with speed u making an acute angle α with horizontal. Moving under gravity it reaches highest point A on parabolic path where its velocity is minimum (equal to $u\cos\alpha$) and finally hits the ground at point B. The total time of motion called time of flight $T = \frac{2u\sin\alpha}{g}$



greatest height attanied $H = AM = \frac{u^2\sin^2\alpha}{2g}$ also $H = \frac{1}{8}gT^2$

range on horizontal plane $R = OB = \frac{u^2\sin(2\alpha)}{g}$ also $\tan\alpha = \frac{4H}{R}$

After time t, it reaches P(x,y) where

$x = ut\cos\alpha$

$y = ut\sin\alpha - \frac{1}{2}gt^2$

the equation of path (trajectory) is $y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$

or $y = \left(x - \frac{x^2}{R}\right)\tan\alpha$

The speed v at P is given by $v^2 = u^2 - 2gy$

The direction of motion θ with horizontal at P is given by

$$\tan\theta = \frac{u\sin\alpha - gt}{u\cos\alpha}$$

Maximum range $R_m = \frac{u^2}{g}$ when $\alpha = 45^o$

For two complementary angles α_1 and α_2 of projection ($\alpha_1 + \alpha_2 = 90^o$) the range are equal $R = 4\sqrt{H_1.H_2}$ where H_1 and H_2 are greatest height attained in these two cases.