MAGNETIC EFFECTS OF CURRENT

INTRODUCTION

The fascinating attractive properties of magnets have been known since ancient times. The word magnet comes from ancient Greek place name Magnesia (the modern town Manisa in Western Turkey), where the natural magnets called lodestones were found.

The fundamental nature of magnetism is the interaction of moving electric charges. Unlike electric forces which act on electric charges whether they are moving or not, magnetic forces act only on moving charges and current carrying wires.

We will describe magnetic forces using the concept of a field. A magnetic field is established by a permanent magnet, by an electric current or by other moving charges. This magnetic field, in turn, exerts forces on other moving charges and current carrying conductors. In this chapter first we study the magnetic forces and torques exerted on moving charges and currents by magnetic fields, then we will see how to calculate the magnetic fields produced by currents and moving charges.

Magnetic Force on a Moving Charge ($\vec{F}_m$)

An unknown electric field can be determined by magnitude and direction of the force on a test charge q_0 at rest. To explore an unknown magnetic field (denoted by $\vec{\mathbf{B}}$), we must measure the magnitude and direction of the force on a moving test charge.

The magnetic force ($\vec{\mathbf{F}}_m$) on a charge q moving with velocity $\vec{\mathbf{v}}$ in a magnetic field $\vec{\mathbf{B}}$ is given, both in magnitude and direction, by

$$\vec{\mathbf{F}}_m = q\left(\vec{\mathbf{v}}\times\vec{\mathbf{B}}\right) \qquad \text{... (i)}$$

Following points are worthnoting regarding the above expression.

(i) The magnitude of $\vec{\mathbf{F}}_m$ is,

$$F_m = Bqv\sin\theta$$

where θ is the angle between $\vec{\mathbf{v}}$ and $\vec{\mathbf{B}}$.

(ii) $\vec{\mathbf{F}}_m$ is zero when,

(a) $B = 0$, i.e., no magnetic field is present,

(b) $q = 0$, i.e., particle is neutral.

(c) $v = 0$, i.e., charged particle is at rest or

(d) $\theta = 0^\circ$ or 180°, i.e., $\vec{\mathbf{v}} \uparrow\uparrow \vec{\mathbf{B}}$ or $\vec{\mathbf{v}} \uparrow\downarrow \vec{\mathbf{B}}$

(iii) $\vec{F}_m$ is maximum at $\theta = 90^o$ and this maximum value is Bqv.

(iv) The units of $\vec{B}$ must be the same as the units of F/qv. Therefore, the SI unit of B is equivalent to $\frac{N-s}{C-m}$.

This unit is called the tesla (abbreviated as T), in honour of Nikola Tesla, the prominent Serbian-American scientist and inventor. Thus,

$$1 \text{ tesla} = 1 \text{ T} = \frac{1\text{N}-\text{s}}{\text{C}-\text{m}} = \frac{1\text{N}}{\text{A}-\text{m}}$$

The CGS unit of $\vec{B}$, the gauss (1 G = 10^{-4} T) is also in common use.

(v) In equation number (i) q is to be substituted with sign. If q is positive magnetic force is along $\vec{v} \times \vec{B}$ and if q is negative magnetic force is in a direction opposite to $\vec{v} \times \vec{B}$.

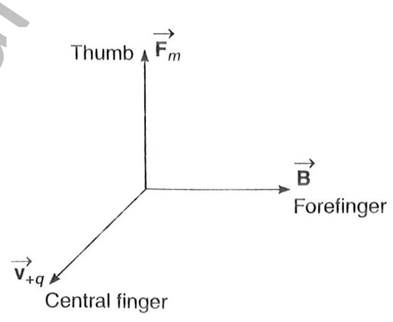
(vi) **Direction of $\vec{F}_m$** : From the property of cross product we can infer that $\vec{F}_m$ is perpendicular to both $\vec{v}$ and $\vec{B}$ or it is perpendicular to the plane formed by $\vec{v}$ and $\vec{B}$. The exact direction of $\vec{F}_m$ can be given by any of the following methods:

(a) **Direction of $\vec{F}_m$** = (sign of q) (direction of $\vec{v} \times \vec{B}$) or, as we stated earlier also

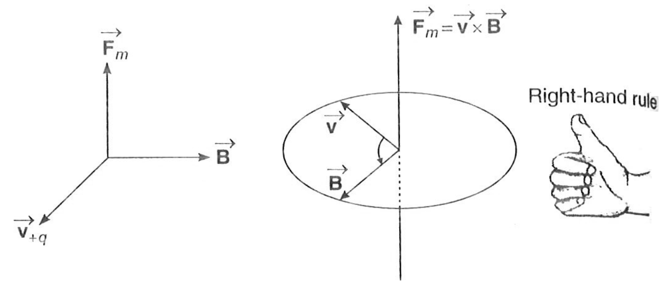
$\vec{F}_m \uparrow\uparrow \vec{v} \times \vec{B}$ if q is positive and

$\vec{F}_m \uparrow\downarrow \vec{v} \times \vec{B}$ if q is negative

(b) **Fleming's left hand rule**: According to this rule, the forefinger, the central finger and the thumb of the left hand are stretched in such a way that they are mutually perpendicular to each other. If the central finger shows the direction of velocity of positive charge ($\vec{v}_{+q}$) and forefinger shows the direction of magnetic field ($\vec{B}$), then the thumb will give the direction of magnetic force ($\vec{F}_m$). If instead of positive charge we have the negative charge, then $\vec{F}_m$ is in opposite direction.



(c) **Right hand rule**: Wrap the fingers of your right hand around the line perpendicular to the plane $\vec{v}$ and $\vec{B}$ as shown in figure. So that they curl around with the sense of rotation from $\vec{v}$ to $\vec{B}$ through the smaller angle between them . Your thumb then points in the direction of the force $\vec{F}_m$ on a positive charge. (Alternatively, the direction of the force $\vec{F}_m$ on a positive charge is the direction in which a right hand thread screw would advance if turned the same way).

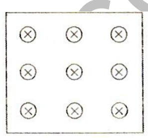


(vii) $\vec{F}_m \perp \vec{v}$ or $\vec{F}_m \perp \frac{d\vec{s}}{dt}$. Therefore, $\vec{F}_m \perp d\vec{s}$ or the work done by the magnetic force in a static magnetic field is zero.

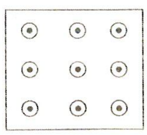
$$W_{\vec{F}_m} = 0$$

So, from work energy theorem KE and hence the speed of the charged particle remains constant in magnetic field. The magnetic force can change the direction only. It cannot increase or decrease the speed or kinetic energy of the particle.

Note: By convention the direction of magnetic field $\vec{B}$ perpendicular to the paper going inwards is shown by $\otimes$ and the direction perpendicular to the paper coming out is shown by $\odot$.



Direction of $\vec{B}$ perpendicular to paper inwards



Direction of $\vec{B}$ perpendicular to paper outwards

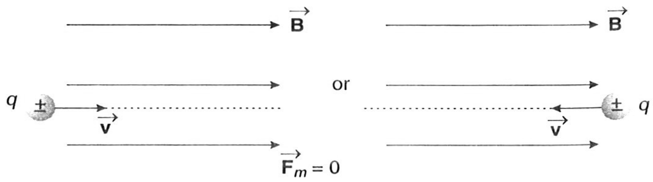
Path of a charged Particle in Uniform Magnetic Field:

The path of a charged particle in uniform magnetic field depends on the angle θ (the angle between $\vec{v}$ and $\vec{B}$). Depending on the different values of θ, following three cases are possible.

Case 1. When θ **is 0° or 180°**

As we have seen in, $\vec{F}_m = 0$, where θ is either 0° or 180°. Hence, path of the charged particle is a straight line (undeviated) when it enters parallel or antiparallel to magnetic field.

Case 2. When $\theta = 90^\circ$



When $\theta = 90^\circ$, the magnetic force is $F_m = Bqv\sin 90^\circ = Bqv$. This magnetic force is perpendicular to the velocity at every instant. Hence, path is a **circle**. The necessary centripetal force is provided by the magnetic force. Hence, if r be the radius of the circle, then

$$\frac{mv^2}{r} = Bqv$$

or $$r = \frac{mv}{Bq}$$

This expression of r can be written in following different ways:

$$\boldsymbol{r = \frac{mv}{Bq} = \frac{P}{Bq} = \frac{\sqrt{2Km}}{Bq} = \frac{\sqrt{2qVm}}{Bq}}$$

Here, P = momentum of particle

K = KE of particle = $\frac{P^2}{2m}$ or

$$P = \sqrt{2Km}$$

We also know that if the charged particle is accelerated by a potential difference of V volts, it acquires a KE given by,

$$K = qV$$

Further, time period of the circular path will be

$$T = \frac{2\pi r}{v} = \frac{2\pi\left(\frac{mv}{Bq}\right)}{v} = \frac{2\pi m}{Bq}$$

or $$\boldsymbol{T = \frac{2\pi m}{Bq}}$$

or the angular speed (ω) of the particle is $\omega = \frac{2\pi}{T} = \frac{Bq}{m}$

$$\omega = \frac{Bq}{m}$$

Frequency of rotation is,

$$f = \frac{1}{T}$$

or $$\boldsymbol{f = \frac{Bq}{2\pi m}}$$

Following points are worthnoting regarding a circular path:

(i) The plane of the circle is perpendicuar to magnetic field. If the magnetic field is along z-direction, the circular path is in x-y plane. The speed of the particle does not change in magnetic field.

Hence, if v_0 be the speed of the particle, then velocity of particle at any instant of time will be,

$$\vec{v} = v_x\hat{i} + v_y\hat{j}$$

where $$v_x^2 + v_y^2 = v_0^2$$

(ii) T, f and ω are independent of v while the radius is directly proportional to v.

Hence, if two, charged particles of equal mass and charge enter in a magnetic field $\vec{\mathbf{B}}$ with different speeds v_1 and $v_2(>v_1)$ at right angles, then.

$$T_1 = T_2$$

but $$r_2 > r_1$$

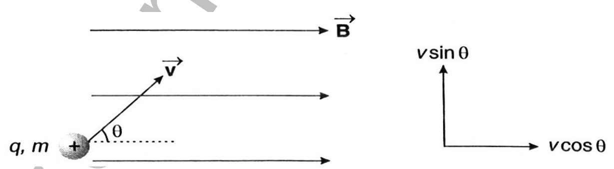
As shown in figure

Note: Charge per unit mass $\frac{q}{m}$ is known as **specific charge**. It is sometimes denoted by α. So, in terms of α, the above formulae can be written as,

$$r = \frac{v}{B\alpha},\ T = \frac{2\pi}{B\alpha},\ t = \frac{B\alpha}{2\pi} \quad \text{and} \quad \omega = B\alpha$$

Case 3: **When θ is other than 0º, 180º, or 90º**

In this case velocity can be resolved in two components, one along $\vec{\mathbf{B}}$ and another perpendicular to $\vec{\mathbf{B}}$. Let the two components by $v_{\parallel}$ and $v_{\perp}$. Then



$$v_{\parallel} = v\cos\theta$$

and $$v_{\perp} = v\sin\theta$$

The component perpendicular to field $(v_{\perp})$ gives a circular path and the component parallel to field $(v_{\parallel})$ gives a straight line path. The resultant path is a helix as shown in figure.

The radius of this helical path is,

$$r = \frac{mv_{\perp}}{Bq} = \frac{mv\sin\theta}{Bq}$$

Time period and frquency do not depend on velocity and so they are given by,

$$T = \frac{2\pi m}{Bq} \quad \text{and} \quad f = \frac{Bq}{2\pi m}$$

There is one more term associated with a helical path, that is **pith (*p*)** of the helical path. Pitch is defined as the distance travelled along magnetic field in one complete cycle. i.e.,

$$p = v_{\parallel} T$$

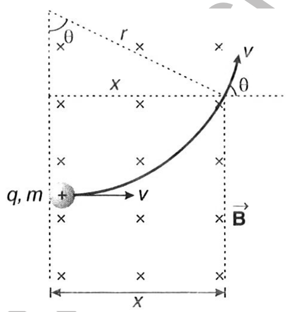
or $$p = (v\cos\theta)\frac{2\pi m}{Bq}$$

$\therefore$ $$p = \frac{2\pi m v\cos\theta}{Bq}$$

Deviation of a charged partcle in magnetic field: Suppose a charged partcle (*q*, *m*) enters a uniform magnetc field $\vec{B}$ at right angles with speed *v* as shown in figure. The magnetic field extends upto a length *x*. The path of the particle is a circle of radius *r*, where

$$r = \frac{mv}{Bq}$$

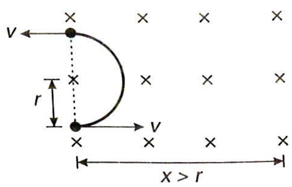
The speed of the particle in magnetic field does not change. But it gets deviated in the magnetic field. The deviation θ can be found in two ways.



(i) After time *t*, deviation will be

$$\theta = \omega t = \left(\frac{Bq}{m}\right)t \qquad \text{as } \omega = \frac{Bq}{m}$$

(ii) In terms of the length of the magnetic field (i.e., when the particle leaves the magnetic field) the deviation will we,



$$\theta = \sin^{-1}\left(\frac{x}{r}\right)$$

But since, $\sin\theta \not> 1$, this relation can be used only when $x \le r$.

For $x > r$, the deviation will be 180° as shown in Fig.